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DETERMINING THE OPTIMUM YIELD OF CONCENTRATED ORE  
IN THE RADIOMETRIC CONCENTRATION OF URANIUM ORES

- USSR -

by Ye. D. Mal'tsev

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## FOREWORD

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DETERMINING THE OPTIMUM YIELD OF CONCENTRATED ORE  
IN THE RADIOMETRIC CONCENTRATION OF URANIUM ORES

Following is a translation of an article by Ye. D. Mal'tsev in the Russian-language periodical Atomnaya energiya (Atomic Energy), Vol. 8, No. 2, Moscow, February 1960, pages 121-126.

(The article gives a method for determining the optimum system for the operation of factories in radiometric ore separation, taking into account the expenditures for geological prospecting  $S_1$ , the yield of ore  $S_2$ , the radiometric concentration  $S_3$ , the hydrometallurgical reduction  $S_4$ , as well as the yield of concentrated ore  $\gamma$ , the content of uranium in the ore  $a$ , the coefficients of the recovery of uranium in the radiometric concentration of the ore  $\xi$ , and the hydrometallurgical reduction  $\xi'$ .)

(For the determination of the minimum cost of uranium salts  $S_m$ , the analytic method was used;  $S_m$  was represented in the form of a continuous function

$$S_m = F(S_1, S_2, S_3, S_4, a, \gamma, \xi, \xi').$$

(According to the theory of approximation, the functions  $\xi = f_1(\gamma)$  and  $\xi' = f_2(a, \gamma, \xi)$  were set up in an analytical form by the method of selected experimental points.

(Since  $S_m = f(\gamma)$  is an experimental function, then equation  $\frac{dS_m}{d\gamma} = 0$  is used to determine the optimum yield of concentrated ore at an ore-separating factory  $\gamma_{opt}$ , corresponding to the minimum cost of the metal.

(An example of the determination of the optimum yield of concentrated ore is presented.)

In connection with the extensive development of the extraction and processing of uranium ores, great importance is attached to the question of reducing to the minimum the cost of extracting uranium salts from ore. One of the most efficient processes significantly reducing the cost of uranium salts is the concentration of the uranium ore in radiometric ore-separating machines. The appropriate adjustment of the ore-separating machines makes it possible to regulate the content of uranium in the tailings of the radiometric concentrating plant and the yield of concentrated ore.

At the present time the uranium content in the tailings of these factories in the majority of cases is established by the level of the uranium content in the tailings of hydrometallurgical plants. In the presence of the technical potentialities of the modern technology of ore processing in hydrometallurgical reduction, this answers the problem of the maximum extraction of uranium, but does not answer the purpose as regards the practicable minimum cost of uranium salts such as may be obtained in the case of the optimum adjustment of radiometric ore-separating machines.

It should be noted that in a number of cases, an increase of 1-1.5% in extraction using radiometric concentration causes an increase of 10-15% in the yield of concentrated ore. This significantly increases the volumes of hydrometallurgical production, the consumption of chemicals, steam, electric power, etc. Thus, with a very small increase in the output of salts, the cost of all of the output produced is increased.

In our opinion, the adjustment of radiometric ore-separating machines, which provides the optimum yield of concentrated ore, will make it possible to reduce the cost of uranium without decreasing the volume of its production.

The cost of uranium salts depends upon expenditures for prospecting for and extracting the ore, upon the cost of concentration and hydrometallurgical reduction, upon the coefficients of extraction during the concentrating and hydrometallurgical processes. The relationship between these factors may be represented in the form of a continuous function and expressed by the equation

$$S_m = \frac{S_1 + S_2 + S_3 + S_4 \gamma}{a \varepsilon \varepsilon'}, \quad (1)$$

where  $S_m$  is the cost of uranium in salts (rubles/kilogram);  $S_1$  is the cost of prospecting for extractable reserves of uranium ore (rubles/ton);  $S_2$  is the cost of extracting the uranium ore (rubles/ton);  $S_3$  is the cost of the radiometric concentration of uranium ore (rubles/ton);  $S_4$  is the cost of transporting the concentrated ore from the mine to the plant and its processing at the hydrometallurgical plant (rubles/ton);  $\gamma$  is the yield of ore by radiometric concentration;  $a$  is the content of uranium in the ore, which goes into radiometric concentration (kilograms/ton);  $\varepsilon$  is the coefficient of the extraction of uranium with radiometric concentration of the ore;  $\varepsilon'$  is the coefficient of extraction of uranium by the processing of the concentrated ore at a hydrometallurgical plant.

In order to determine the optimum cost of uranium in the final product from equation (1), it is necessary to establish the relationship between  $\varepsilon$ ,  $\varepsilon'$ , and  $\gamma$ :

$$\varepsilon = f_1(\gamma), \quad \varepsilon' = f_2(\gamma).$$

If in a system of rectangular coordinates one plots points obtained experimentally (along the abscissa axis -- the yield of ore by radiometric concentration, and along the ordinate axis -- the coefficient of extraction, corresponding to this yield), such a graph is obtained as is presented in Figure 1.

Obviously, for each type of ore, depending upon the physical-mechanical properties, difference, the system of radiometric concentration used, and the type of concentrating machine, its own curve can be constructed; and on the whole for the process of radiometric concentration, a set of curves.

Arising from the essence of the concentration process, the equation expressing the relationship between  $\xi$  and  $\gamma$  must satisfy the following conditions: when  $\gamma = 0$   $\xi = 0$ , i.e., zero extraction corresponds to a zero yield of concentrated ore; when  $\gamma = 1$   $\xi = 1$ , i.e., a 100% extraction corresponds to a 100% yield of ore; in addition, the curve  $\xi = f(\gamma)$  must go through two selected characteristic points with coordinates  $\gamma_1, \xi_1$ , and  $\gamma_2, \xi_2$ , obtained experimentally.

In a particular case, the equation  $\xi = f_1(\gamma)$  must be transposed into an equation of the bisectrix of the coordinate angle  $\xi = \gamma$ , representing the relationship between  $\xi$  and  $\gamma$  for ores, which because of their own properties do not lend themselves to concentration.

Investigations indicated that the following form of the equation, which satisfies the indicated set of curves, is the most suitable:

$$\xi = \delta \gamma^e - \frac{\gamma}{1 + \gamma^v}, \quad (2)$$

where  $\delta, e, v$  are the coefficients characterizing the physical-mechanical properties of the ore and the conditions of its concentration.

If we take  $\gamma = 1$  and  $\xi = 1$ , then we shall get the first equation for the determination of the coefficient :

$$\xi = \delta \cdot 1^e - \frac{1}{1 + 1^v} = \delta - 0.5 = 1,$$

hence  $\delta = 1.5$ .

Obtained by the experimental method, the two points on the curve, i.e., the two values of the yield of the concentrated ore and the coefficients of extraction corresponding to them, make it possible to write the equations

$$\xi = f_1(\gamma) = \frac{\lg \left( \xi_1 + \frac{\gamma_1}{1 + \gamma_1^v} \right) - \lg 1.5}{\lg \gamma_1} : \quad (3)$$

$$\epsilon = f_2(v) = \frac{\lg \left( \epsilon_2 + \frac{\gamma_2}{1 + \gamma_2^v} \right) - \lg 1.5}{\lg \gamma_2} \quad (4)$$

From equations (3) and (4) by the graphoanalytic method we determine the values  $v$  and  $\epsilon$ , having constructed curves for this. At the intersection of these curves we shall obtain the unknown values.

It is necessary to have more than two experimental points, in order to characterize the process of concentration by means of equation (2). In this case, for the determination of constant coefficients  $\epsilon$  and  $v$ , the most characteristic points on the curve, constructed on experimental data, may be selected.

In its essence, the hydrometallurgical process is analogous to the concentration process, because in both cases the concentration of the useful component occurs. Therefore the relationship between the extraction in the hydrometallurgical reduction  $\epsilon'$  and the content of uranium in the ore  $\beta$ , coming for hydrometallurgical processing, may also be established according to equation (2). However, it should be noted that it is not necessary to investigate the relationship between  $\epsilon'$  and  $\beta$  within such wide limits (from 0 to 100%) as was done in the investigation of the relationship between the extraction coefficient and the concentration coefficient of the ore in radiometric reduction. Let us limit ourselves to the study of the relationship between  $\epsilon'$  and  $\beta$  in the range of change of  $\beta$  from  $a$  to  $\beta_0$ , which corresponds to a specific given  $\gamma_0$ .

The quantity of the metal, which will remain in the concentrated ore after radiometric reduction, will be equal to  $a\epsilon$ ; then the content in the concentrated ore  $\beta$  is determined by the expression

$$\beta = \frac{a\epsilon}{\gamma} = a \left( \delta \gamma^{\epsilon-1} - \frac{1}{1 + \gamma^v} \right) = f_0(\gamma). \quad (5)$$

Provided that the relationship between  $\epsilon'$  and  $\beta$  will be investigated in the interval from  $a$  to  $\beta_0$ , corresponding to  $\gamma_0$ , we shall obtain the equation for the determination of the extreme  $\beta_0$ :

$$\beta_0 = a \left( \delta \gamma_0^{\epsilon-1} - \frac{1}{1 + \gamma_0^v} \right). \quad (6)$$

The change of the extraction coefficient in hydrometallurgical reduction is represented by the graph presented in Figure 2.

As has been indicated, the relationship between  $\epsilon'$  and  $\beta$  we take in the form

$$\epsilon' = \delta_1 \beta^{\epsilon-1} - \frac{\beta}{1 + \beta^v}. \quad (7)$$

The coefficients  $\delta_1, e_1, v_1$  are determined according to three experimental points

$$\delta_1 = \frac{1}{a e_1} \left( \varepsilon_1' + \frac{a}{1 + a v_1} \right); \quad (8)$$

$$e_1 = f_1(v_1) = \frac{\lg \left( \varepsilon_1' + \frac{a}{1 + a v_1} \right) - \lg \left( \varepsilon_2' + \frac{\beta_1}{1 + \beta_1 v_1} \right)}{\lg a - \lg \beta_1}; \quad (9)$$

$$e_1 = f_2(v_1) = \frac{\lg \left( \varepsilon_1' + \frac{a}{1 + a v_1} \right) - \lg \left( \varepsilon_0' - \frac{\beta_0}{1 + \beta_0 v_1} \right)}{\lg a - \lg \beta_0}. \quad (10)$$

Having constructed the curves according to equations (9) and (10), we shall obtain at their intersection the unknown values  $e_1$  and  $v_1$ ; we shall determine the value  $\delta_1$  from equation (8).

Since, according to equation (5)  $\beta = f_0(\gamma)$ , the relationship between  $\gamma$  and  $\varepsilon'$  in the definitive form will be expressed by the equation

$$\varepsilon' = \delta_1 \sqrt{f_0(\gamma)}^{\varepsilon_1} - \frac{f_0(\gamma)}{1 + \sqrt{f_0(\gamma)}^{\varepsilon_1}}. \quad (11)$$

For the optimum value  $\gamma$  let us substitute the values obtained for  $\varepsilon$  and  $\varepsilon'$  in equation (1):

$$S_m = \frac{S_1 + S_2 + S_3 + S_4 \gamma}{a \sqrt{\delta_1}^{\varepsilon} - \frac{\gamma}{1 + \gamma \sqrt{\delta_1}^{\varepsilon}} \sqrt{\delta_1} \sqrt{f_0(\gamma)}^{\varepsilon_1} - \frac{f_0(\gamma)}{1 + \sqrt{f_0(\gamma)}^{\varepsilon_1}}}. \quad (12)$$

The graph of equation (12) is an experimental curve which has its minimum at the point corresponding to the least cost of uranium and the optimum yield of concentrated ore.

Let us determine the position  $S_m$  of the minimum  $\gamma_{opt}$ , after having taken the first derivative from the cost of uranium  $S_m$  according to  $\gamma$  and having made it equal to zero.

This equation in regard to  $\gamma$  may be solved by the graphoanalytic method, after having divided it into two equations:

$$y = \varepsilon \varepsilon' S_4; \quad (13)$$

$$y = \left( \varepsilon' \frac{d\varepsilon}{d\gamma} + \varepsilon \frac{d\varepsilon'}{d\gamma} \right) (S_1 + S_2 + S_3 + S_4 \gamma). \quad (14)$$

The value  $\frac{d\varepsilon}{d\gamma}$  is determined according to equation

$$\frac{d\varepsilon}{d\gamma} = \varepsilon \delta \gamma^{\varepsilon-1} - \frac{\gamma^v(1-v) + 1}{(1+\gamma^v)^2}. \quad (15)$$

The value  $\frac{d\varepsilon'}{d\gamma}$  is found by differentiation of the equation (7):

$$\frac{d\varepsilon'}{d\gamma} = \frac{d\beta}{d\gamma} \left[ \delta_1 \varepsilon_1 \beta^{\varepsilon_1-1} - \frac{1 + \beta^{v_1}(1-v_1)}{(1+\beta^{v_1})^2} \right]. \quad (16)$$

We shall determine the value  $\frac{d\beta}{d\gamma}$  after having differentiated equation (5):

$$\frac{d\beta}{d\gamma} = a \delta (\varepsilon - 1) \gamma^{\varepsilon-2} + \frac{\gamma^{v-1}}{(1+\gamma^v)^2}. \quad (17)$$

Since before radiometric concentration the ore is separated by being sifted into classes of coarseness, each class being individually submitted to concentration, the relationship between  $\varepsilon$  and  $\gamma$  for each class may be characterized by its own constant coefficients  $\varepsilon$  and  $\gamma$ . It is necessary to carry out for all classes of coarseness the cited calculation for the determination of the optimum system of operation of the machine, determining for each class of coarseness its optimum yield of concentrated ore and the minimum cost of the metal. Thus, after having separated the total cost of the ore, including expenditures for geological prospecting work, the cost of the ore of each class of coarseness should be determined proportionally to the distribution of the metal by classes.

The determination of the optimum yield of concentrated ore of each class of coarseness should be carried out according to the equation

$$S_m = \frac{S_k + S_3 + S_4 \gamma}{a \varepsilon \varepsilon'},$$

where  $S_k$  is the cost per ton of ore of a given class of coarseness, including expenditures for geological prospecting.

Let us assume that the ore is separated by being sifted into  $i$  classes with an ore quantity in each class of  $Q_1, Q_2 \dots Q_i$  and with a metal content of  $a_1, a_2 \dots a_i$ . In this case the total optimum cost of the metal as a whole for all of the ore will be determined by the expression

$$S_m = \frac{Q_1 a_1 S_{m1} + Q_2 a_2 S_{m2} + \dots + Q_i a_i S_{mi}}{Q_1 a_1 + Q_2 a_2 + \dots + Q_i a_i}. \quad (18)$$



If one of the classes, due to its structure, does not lend itself to radiometric concentration, then the cost of the metal obtained from the ore of this class is determined by equation (1) with  $S_3 = 0$  and  $\gamma = 1$ .

Let us cite an example of calculation for one class:  $S_1 = 20$  rubles/ton;  $S_2 = 60$  rubles/ton;  $S_3 = 10$  rubles/ton;  $S_4 = 150$  rubles/ton;  $a = 1.0$  kilogram/ton = 0.1%.

The data were obtained by the experimental method:

For the ore's ability to be concentrated:

$\gamma_1$	1.0	$\epsilon_1$	1.0
$\gamma_2$	0.62	$\epsilon_2$	0.94
$\gamma_3$	0.25	$\epsilon_3$	0.75

According to the hydrometallurgical process

$\beta_1$	0.1	$\epsilon'_1$	0.900
$\beta_2$	0.151	$\epsilon'_2$	0.914
$\beta_3$	0.300	$\epsilon'_3$	0.939

The coordinates  $\epsilon$  and  $\gamma$ , calculated according to equations (3) and (4), are presented in Table 1. For the determination of values  $\epsilon$  and  $\gamma$  a graph (Figure 3) is constructed.

TABLE 1

COORDINATES OF POINTS  $\epsilon$  AND  $\gamma$

$\gamma$	$\epsilon = f_1(\gamma)$	$\epsilon = f_2(\gamma)$
+ 2.0	0.159	0.304
+ 1.0	0.264	0.329
0.0	0.380	0.389
- 0.5	0.447	0.424
- 1.0	0.500	0.454
- 3.0	0.727	0.496

Thus equation (2) will assume the form

$$\varepsilon = 1.5\gamma^{0.40} + \frac{\gamma}{1 + \gamma^{-0.12}}.$$

The graph of equation (2) is presented in Figure 4.

The values  $\varepsilon_1$  and  $\gamma_1$ , determined according to equations (9) and (10), are presented in Table 2.

TABLE 2

COORDINATES OF POINTS  $\varepsilon_1$  AND  $v_1$

$v_1$	$\varepsilon_1 = f_1(v_1)$	$\varepsilon_1 = f_2(v_1)$
+ $\infty$	0.151	0.194
+ 1.0	0.123	0.151
0.0	0.101	0.123
- 1.0	0.062	0.092
- 3.0	0.040	0.046
- 5.3	0.038	0.038
- $\infty$	0.038	0.038

The graphs  $\varepsilon_1 = f_1(v_1)$  and  $\varepsilon_1 = f_2(v_1)$  are presented in Figure 5.

The value  $\delta_1$ , calculated according to equation (8), is equal to 0.987. Then equation (7) will take the form

$$\varepsilon' = 0.982\beta^{0.04} - \frac{\beta}{1 + \beta^{-5.0}} = f(\beta).$$

The graph of the equation  $f(\beta)$  is presented in Figure 6.

The value of optimum yield of concentrated ore is determined according to equations (13) and (14). The coordinates  $\gamma$  and  $y$  are presented in Table 3. The graph  $y = f_1(\gamma)$  and  $y = f_2(\gamma)$  is presented in Figure 7.

TABLE 3

COORDINATES  $\gamma$  AND  $y$ 

<u>According to equation (13)</u>		<u>According to equation (14)</u>	
$\gamma$	$y$	$\gamma$	$y$
0.1	80.3	0.1	195.0
0.3	110.5	0.3	92.4
0.5	124.0	0.5	58.4
0.7	131.5	0.7	35.1
0.9	134.5	0.9	16.6
1.0	135.0	1.0	7.2

Let us calculate according to equation (12) the coordinates of the points and let us construct the curve  $S_m = f(\gamma)$ :

$\gamma$	$S_m$
0.1	196.5
0.3	183.0
0.5	199.5
0.7	222.0
0.9	251.0
1.0	267.0

Graph  $S_m = f(\gamma)$  is presented in Figure 8. As can be seen by this figure, there is a clearly expressed range of optimum yields of concentrated ore, which is characterized by the minimum costs of uranium salts.

In this case, if the economic parameters of the extraction and processing of ore make it possible to obtain a uranium cost in the final product significantly lower than its average cost in industry, then it seems to us expedient to depart somewhat from the optimum in favor of an increased yield of concentrated ore  $\gamma$  and to thus increase the extraction and yield of uranium in the final product. However, the degree of departure from the optimum should be regulated so that the cost of obtaining an additional kilogram of metal will not exceed the average cost of metal in industry.

# FIGURE APPENDIX

$$S_m = \frac{S_1 + S_2 + S_3 + S_4 \gamma}{a \cdot E'}$$

Figure 1. The relationship between the yield of concentrated ore  $\gamma$  and the extraction coefficient of uranium  $E$ .

$$E' = E - \frac{\gamma}{1 + \gamma^v}$$

Figure 2. The variation of the extraction coefficient  $E'$  in hydrometallurgical reduction in relation to the content of uranium in the ore  $\beta$ .

$$Q = f_1(v) = \frac{\lg \left( e_1 + \frac{y_1}{1 + y_1^v} \right) - \lg 1.5}{\lg y_1}$$

Figure 3. The determination of coefficients  $v$  and  $Q$ .

$$Q = f_2(v) = \frac{\lg \left( e_2 + \frac{y_2}{1 + y_2^v} \right) - \lg 1.5}{\lg y_2}$$

Figure 4. The relationship of the extraction of the metal in the rich product of radiometric separation to the yield of this product.

$$\beta = \frac{a\gamma}{\gamma} = a \left( \delta \gamma^{q-1} - \frac{1}{1 + \gamma^q} \right) = f_0(\gamma).$$

Figure 5. Determination of coefficients  $v_1$  and  $q_1$ .

$$\beta_0 = a \left( \delta \gamma_0^{q-1} - \frac{1}{1 + \gamma_0^q} \right).$$

Figure 6. Relationship of the extraction of the metal in the concentrate of hydrometallurgical reduction to the content of the metal in the ore (rich product of radiometric separation).

$$e' = \delta_1 p^0_1 - \frac{\beta}{1 + \beta v_1}$$

Figure 7. Determination of the optimum yield of the rich product of radiometric separation.

$$\delta_1 = \frac{1}{a v_1} \left( r_1 - \frac{a}{1 + a v_1} \right);$$

Figure 8. Relationship of the cost of the metal in the final product of hydrometallurgical reduction to the yield of the rich product of radiometric separation.

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